Linear functions

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Linear functions

Definition

A *linear function* is a function whose graph is a straight line. Also, a linear function is one that can be written in the form $y = mx + b$. Linear functions describe increase/decrease at a fixed amount per unit change in input.

Examples:

- Temperature in $^\circ C$ as a function of $^\circ F$: $C = 5/9\,(F - 32)$
- Value in Euros E as a function of dollars D: $E = 0.76D$
- Distance D covered in t hours when driving at speed r: $D = rt$
- Okun's law relating % increase y in annual U.S. production during a year when unemployment changes by u %: $y = 3.5 - 2u$.

Parameters of linear functions
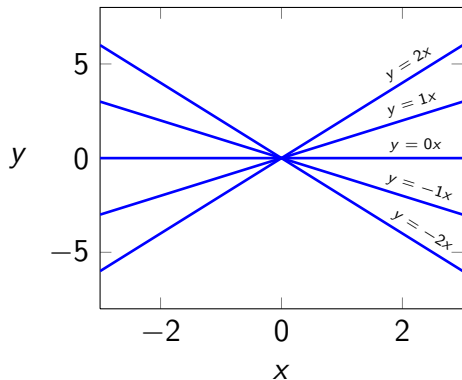
For the linear function $y = mx + b$

- b is *y-intercept*, specifies value of y when $x = 0$.
- Increasing/decreasing b moves graph of line up/down.
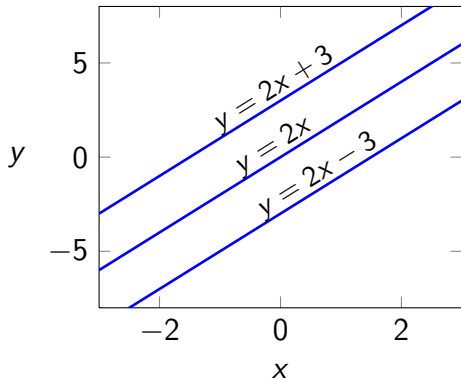
Parameters of linear functions

For the linear function $y = mx + b$

- m is *slope*, specifies how much y changes for given change in x.
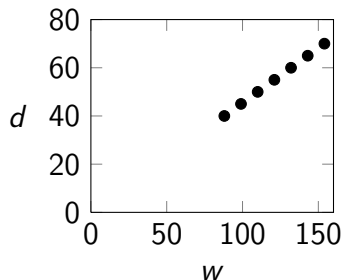- Increasing/decreasing m rotates line counter/clockwise.

More on slope

Since a line only has one slope, we can assess if data is linear by checking if the slope is the same between all points.

Example: Recommended dosage of the antibiotic Tobramycin as a function of body weight.

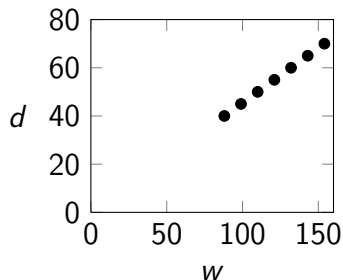| Weight w (lbs)	88	99	110	121	132	143	154
Dosage d (mg)	40	45	50	55	60	65	70

$$
\begin{aligned}
m &= \frac{\Delta d}{\Delta w} \\
&= \frac{45 - 40}{99 - 88} \\
&= \frac{5}{11} \ \frac{mg}{lb}
\end{aligned}
$$

More on slope

Since a line only has one slope, we can assess if data is linear by checking if the slope is the same between all points.

Example: Recommended dosage of the antibiotic Tobramycin as a function of body weight.

Weight w (*lbs*)	88	99	110	121	132	143	154
Dosage d (*mg*)	40	45	50	55	60	65	70

$$
\begin{aligned}
m &= \frac{\Delta d}{\Delta w} \\
&= \frac{65 - 50}{143 - 110} \\
&= \frac{15}{33} = \frac{5}{11} \; \frac{mg}{lb}
\end{aligned}
$$

Finishing equation of line

Once the slope is known, find y-intercept b by plugging in data.

Example: Recommended dosage of the antibiotic Tobramycin as a function of body weight.

Weight w (lbs)	88	99	110	121	132	143	154
Dosage d (mg)	40	45	50	55	60	65	70

$$y = mx + b \quad \text{with } m = 5/11$$

Choose (121,55) to substitute

$$
\begin{aligned}
y &= 5/11\,x + b \\
55 &= 5/11 \cdot 121 + b \\
b &= 0
\end{aligned}
$$

$$y = \frac{5}{11}x$$

Reflect

Ask yourself: "Can I..."

- Explain linear functions in words, equations, and graphs?
- Describe the effects of varying parameters in linear functions?
- Assess whether given data is linear and, if so, model it?