

Introduction to functions

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Welcome to functions

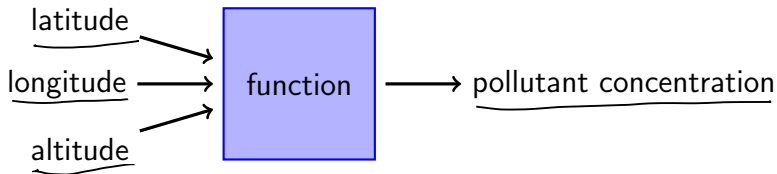
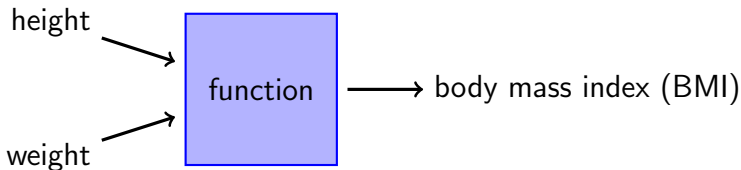
- Key goal: mathematical modeling.
- A *model* is a (simplified) representation of something real.
- Functions are a basic type of mathematical model.
- A function is not necessarily a formula!

Definition

A *function* is a rule taking one or more inputs (called *independent variables*) and assigning a definite output (called the *dependent variable*).



Examples of functions



Representing functions

Remember: a function is a rule that accepts inputs and gives a definite output, and this need not be a formula.

There are (at least) four ways to represent functions:

- Table of data (numerical)
- Graph/picture
- Equation
- Verbal description

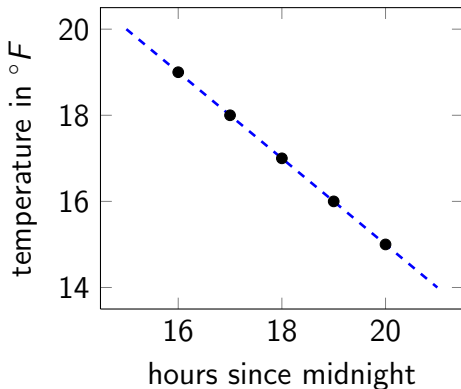
Example: representing functions (table)

Consider the temperature on a particular day in St. Paul, MN.

Hours since midnight	$^{\circ}F$
16	19
17	18
18	17
19	16
20	15

Example: representing functions (graph)

Consider the temperature on a particular day in St. Paul, MN.



Example: representing functions (equation)

Consider the temperature on a particular day in St. Paul, MN.

Let t = time in hours since midnight and let y = temperature in $^{\circ}F$.
Then

$$y = 35 - t.$$

Or, to make the independent and dependent variables even more explicit,


$$y(t) = 35 - t.$$

Example: representing functions (words)

Consider the temperature on a particular day in St. Paul, MN.

To find the temperature at time t hours since midnight, just subtract t from 35.

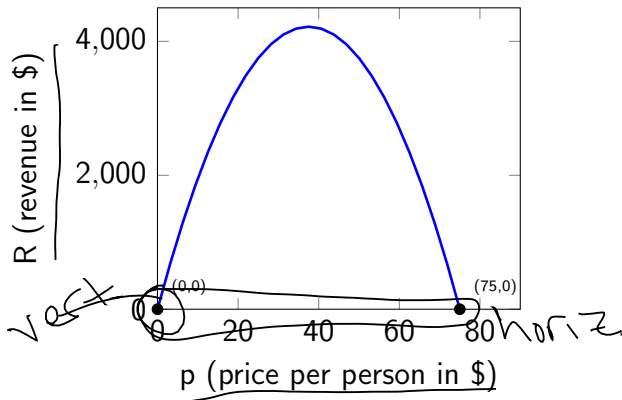
We can also note that the temperature is $19^\circ F$ at 4 p.m. and drops at one degree per hour.



Basic aspects of functions

Intercepts (horiz. and vert.) are where f crosses axis.

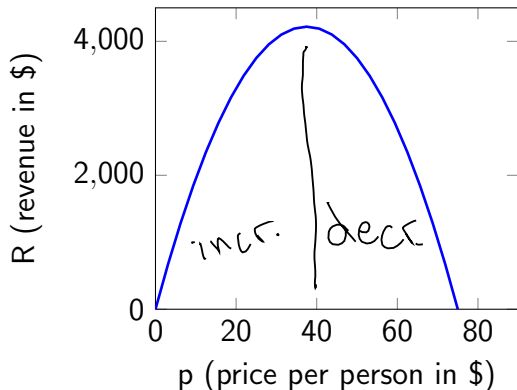
$$R(p) = 225p - 3p^2$$



Basic aspects of functions

Increasing/decreasing: what f does as an indep. variable *increases*.

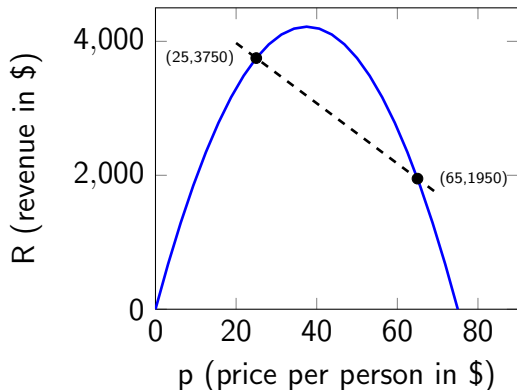
$$R(p) = 225p - 3p^2$$



Basic aspects of functions

Average rate of change is slope of line connecting two points on f .

$$R(p) = 225p - 3p^2$$

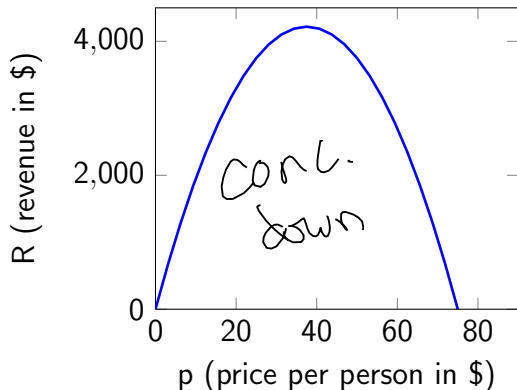


$$\begin{aligned} \frac{\Delta R}{\Delta p} &= \frac{1950 - 3750}{65 - 25} \\ &= -45 \frac{\$ \text{ revenue}}{\$ \text{ price}} \end{aligned}$$

Basic aspects of functions

Concavity tells if f curves up or down.

$$R(p) = 225p - 3p^2$$



Conc.
UP.



incr.
C.d.



decr.
C.u.



incr.
C.u.

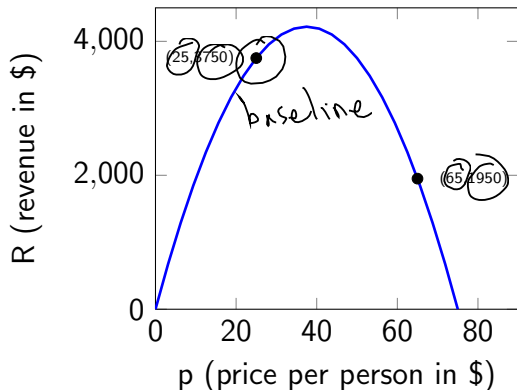


decr.
C.d.

Basic aspects of functions

Relative chg. is chg. in f from baseline val., divided by baseline val.

$$R(p) = 225p - 3p^2$$



Relative change from
 $p = 25$ to $p = 65$ is

$$\begin{aligned} & \frac{\text{change in } R}{\text{initial } R} \\ &= \frac{1950 - 3750}{3750} \\ &= -0.48 \rightarrow 48\% \text{ decr.} \end{aligned}$$

Reflect

Ask yourself: “Can I...”

- Explain what a function is?
- Identify and explain independent and dependent variables?
- Recognize functions in a variety of forms?
- Find a function's intercept(s)?
- Identify regions where a function is increasing/decreasing?
- Compute the average rate of change between two points on a function?
- Determine where a function is concave up/down?
- Compute the relative change between two points on a function?